The Topological and Electrical Structure of Power Grids

P. Hines
U. of Vermont
paul.hines@uvm.edu

S. Blumsack
Pennsylvania State U.
blumsack@psu.edu

E. Cotilla Sanchez
U. of Vermont
jcotilla@uvm.edu

C. Barrows
Pennsylvania State U.
cpb155@psu.edu

Abstract

Numerous recent papers have found important relationships between network structure and risks within networks. These results indicate that network structure can dramatically affect the relative effectiveness of risk identification and mitigation methods. With this in mind this paper provides a comparative analysis of the topological and electrical structure of the IEEE 300 bus and the Eastern United States power grids. Specifically we compare the topology of these grids with that of random [1], preferential-attachment [2] and small-world [3] networks of equivalent sizes and find that power grids differ substantially from these abstract models in degree distribution, clustering, diameter and assortativity, and thus conclude that these abstract models do not provide substantial utility for modeling power grids. To better represent the topological properties of power grids we introduce a new graph generating algorithm, the minimum distance graph, that produces networks with properties that more nearly match those of known power grids. While these topological comparisons are useful, they do not account for the physical laws that govern flows in electricity networks. To elucidate the electrical structure of power grids, we propose a new method for representing electrical structure as a weighted graph. This analogous representation is based on electrical distance rather than topological connections. A comparison of these two representations of the test power grids reveals dramatic differences between the electrical and topological structure of electrical power systems.

1. Introduction

Recent research in complex networks has elucidated strong links between the structure of a network and risks within that network. Albert et al. [4] found that scale-free networks [2], which are characterized by strongly heterogeneous (power-law) node connectivity (degree), are uniquely robust to random failures but vulnerable to directed attacks. However graphs with exponential degree distributions, such as the random graph [1], were found to be equally vulnerable to random failures and directed attacks. Extending these results, Holme and Kim [5] found that optimal attack strategies change as a network degrades. Studies of failures in the Internet [6] and commercial air traffic networks [7], provide empirical evidence for these theoretical findings. Nearly comprehensive reviews of the complex networks literature are available in [8, 9].

The relationship between structure and performance in networks has implications for managing and mitigating risks related to network attacks or failures. Garber [6] describes strategies for reducing risks associated with the scale-free structure of the Internet based on increasing redundancy at the network hubs. Carley et al. [10, 11] relate organizational structure to effectiveness in covert intelligence networks. Dodds and Watts [12] model formal and informal organizational network ties finding that efficiency trades off against system resilience best for multi-scale, hierarchical structures. Eubank et al. [13] find evidence that the graph of locations in urban social networks is scale-free and shows that small-pox risk can be effectively mitigated by placing sensors at the hub locations. Watts et al. [14] show that broad distributions of disease outbreaks arise in a simple hierarchical meta-population model, and that limiting travel distance before frequency may control spreading. Miller and Hyman [15] find that a disease vaccination strategy based on connectivity, particularly the number of locations or persons that an agent regularly visits, can be significantly more effective than random vaccination.

Given their size, complexity and importance to modern economies, it is not surprising that power grids have received a fair amount of attention in the literature on network science and complex systems. Watts and Strogatz [3] measure the characteristic path length and clustering in power grids, and find similarities to "Small-World" network models. A number of studies measure the degree distribution of various power grids with some reporting exponential [16, 17] and others reporting power-law/scale-free degree distributions [18, 19]. That studies of different power grids in different countries or regions yield different topological structures is not necessarily surprising. Much more surprising is that different analyses of identical grids (the transmission system in the Western U.S.) have yielded different structural results [18, 17]. This paper clarifies this uncertainty by showing that, at least for the IEEE 300 bus network and the US Eastern Interconnect, an exponential degree distribution is a better fit to the data than a power-law distribution. In addition to reporting an exponential degree distribution [17] also report a power-law in the topological betweenness of nodes in power grids, which is proposed as a potential explanation for the power-law that is observed in the frequency of blackouts in the US [20] and Sweden [21]. Holmgren [22] compares the Western US power grid and the Nordic grid using the topological attack/failure model in [4], and finds that the Nordic grid has more of a fat-tailed degree distribution structure. He finds that the Nordic grid has a more fat-tailed degree distribution than the Western US and provides some recommendations for increasing the robustness of power grids.

While they do provide insight into network structure, these initial studies focus on physical topology and largely neglect the equations that govern flows in power networks, namely Ohm's and Kirchhoff's laws. To address this deficiency, Hines and Blumsack [23] describe a measure of Electrical Centrality for power grids. Bompard et al. [24, 25] combine topological models with DC load flow models and propose new measures that can be useful in identifying critical components. Blumsack et al. [26] notes relationships between Wheatstone structures within power grids, reliability and efficiency. Relatedly, Wang et al. [27] present a method for generating synthetic power grids.

The field of power systems has a long history of vulnerability assessment. Automatic contingency selection and ranking methods based on sensitivities (linear or nonlinear) [28, 29] have been industry standard procedure for quickly identifying vulnerabilities. Other approaches have been to simulate specific events on the transmission to detect vulnerabilities (a summary of these approaches can be found in [30]), as well as applying traditional techniques of probabilistic risk assessment [31, 32]. The challenge with methods requiring system simulations is overcoming high dimensionality; in a large system, there are an enormous number of different combinations of events that could represent a system vulnerability [33]. Only recently have researchers in the power systems field begun to take complex-systems approaches to understand the nature of vulnerability [20, 34], and to incorporate structural information from the network in the detection and mitigation of failures [35, 36, 37].

1.1. Goals and outline of this paper

This paper aims to fill a number of gaps in this existing research on the structure of power grids, and to provide a foundation for new tools for vulnerability assessment. The data used in this study include a standard test case, the IEEE 300 bus network, and a 49,907 bus model of the US Eastern Interconnect (EI). The EI model is substantially more detailed and accurate than models that have been used in past structural studies of the North American grid.

Section 2 provides a topological analysis of the two networks, showing how power grids are topologically similar to and different from random [1], small-world [3], and preferential attachment [2] graphs. In Section 3 we propose a new algorithm for creating synthetic networks with properties that are similar to power grids, relative to these existing models. Section 4 proposes a new method for studying power grids as complex networks, based primarily on electrical, rather than physical, structure. Section 5 discusses some implications of these results for identifying vulnerabilities.

2. The topology of power grids

As previously noted, the existing literature shows some disagreement over the topological structure of power networks. Some report a power-law degree distribution whereas others argue that an exponential fit is superior. This section aims to clarify this and other uncertainties regarding the structure of power grids using a larger, more accurate model of the Eastern US system than has been used in past studies. To further show how power grids are similar to and different from common abstract network models, we compare the topology of the US Eastern Interconnect (EI) to similarly sized synthetic graphs. A similar, but less complete, comparison is available in [27].

In this section we represent each test network as an undirected, unweighted graph with n vertices/nodes and m links/edges. For the power grid models all buses, whether generator, load, or pass-through, are treated equally. For the power grids, n is the number of buses and m is the number of connected bus pairs. Note that the links can represent a set of parallel transformers or transmission lines, which means that m may be slightly smaller than the number of branches in the power system model. The set of all vertices and edges in each graph, G, is $\{N, M\}$. The adjacency matrix for graph G_X is A_X, with elements a_{ij}.

The following sub-sections describe the power grid data, the synthetic network models and the metrics that are used for comparison purposes.

2.1. Power grid data

In this paper we look at two power grid data sets: a standard test system (the IEEE 300 bus system) and a large model of US Eastern Interconnect. The IEEE 300 bus system (see Fig. 1) is available from [38]. The system

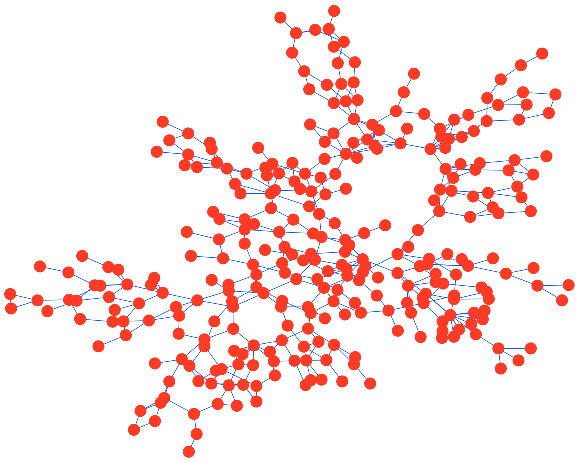

Figure 1. Topology of the IEEE 300 node system

has 411 branches, and average degree ($< k >$) of 2.74. Two of the branches are parallel lines, so the graph size is: $|G_{300}| = \{300, 409\}$. The Eastern Interconnect (EI) data come from a NERC planning model for 2012. The NERC EI planning models are known as MMWG (Multiregional Modeling Working Group) cases, and are classified as "Critical Energy Infrastructure Information" by the US Department of Energy. The authors have obtained permission to use these data for research purposes. The EI model has 49,907 buses, though in our model 310 of these are isolated from the larger sub-components. After removing the isolates and parallel branches, we obtain a graph (G_{EI}) with 49,597 vertices, 62,985 links and an average degree $< k >= 2.54$.

2.2. Synthetic networks

To show how power grids differ from other network structures we generate three graphs with similar sizes to the IEEE 300 and EI graphs: A small-world [3], preferential attachment (PA) [2], and a random graph [1]. Each graph is generated to have the same number of nodes and nearly the same number of branches as the power grid.

The random graph is generated using the standard algorithm [1, 4] with a fixed number of nodes and links.

To generate a preferential attachment/scale-free (PA) graph with roughly n nodes and m links we modify the algorithm described in [2] somewhat. For each new node a we initially add one link between a and an existing node b using the standard roulette wheel method. Specifically node b is selected randomly from the probability distribution $P_{a \to b} = k_b / \sum_c k_c$. After adding this initial link a second is added with probability $m/n - 1$. Thus the addition of each new node results in an average of $1 + (m/n - 1) = m/n$ new links, producing a preferential attachment graph with n nodes and roughly m links.

The small-world model is argued in [3] to bear some resemblance to power grids. To test this we generate a regular lattice with n nodes and approximately m links. The initial links in the regular lattice are created in roughly the same way as the modified PA graph above. With each new node, a link is created to a neighboring node (for node a, the first link is to $a - 1$). A second link is then created to node $a - 2$ with probability $m/n - 1$, thus giving approximately m links in total. Note that $a - 1$ and $a - 2$ need to be adjusted for the first two nodes in the graph. After generating the regular lattice in this manner random re-wiring proceeds according to the method described in [3] until the diameter is approximately the same as the corresponding power grid.

2.3. Measures of graph structure

There are many useful statistical measures for graphs. Among the most useful are degree distribution [2], characteristic path length [3], graph diameter [8], clustering coefficient [8], and degree assortativity [2]. These measures provide a useful set of statistics for comparing power grids with other graph structures.

The probability mass function (pmf) for node connectivity, or degree distribution, describes the diversity of connectivity in a graph. While the measure has a long history, recent results showing that many real networks have a power-law degree distribution (so-called scale-free networks [2]) has emphasized the value of the measure. The extent to which the degree distribution is fat-tailed indicates the number of hubs within the network. The degree of node i in a graph with adjacency matrix A is:

$$k_i = \sum_{j=1}^{n} a_{ij} \quad (1)$$

and the degree distribution is $\Pr(k = x) = n_k/n$, where n_k is the number of nodes of degree k. Often it is more convenient to work with the complementary cumulative distribution function (ccdf). For scale free networks, where the power-law tail starts at $x_{\min}$, the ccdf is:

$$\Pr(k \geq x) = \frac{x_{\min}^{\alpha}}{x^{\alpha+1}}$$

If the degree distribution is exponential, as found in random graphs, a minimum value Weibull distribution provides a better fit to the data:

$$\Pr(k \geq x) = e^{-\left(\frac{x - x_{\min}}{\lambda}\right)^{\beta}}$$

Many real networks show substantial clustering among nodes. Watts and Strogatz [3] report that the network of collaborations among film actors and the neural structure of the *C. elegant* worm show substantial clustering, whereas clustering in the Western US power grid is smaller. In this paper we use the clustering coefficient described in [3]:

$$C = \frac{1}{n}\sum_{i=1}^{n} c_i \tag{2}$$

where the clustering of node i (c_i) is

$$c_i = \frac{e_i}{(k_i(k_i-1))/2} \tag{3}$$

and e_i is the number of edges within the cluster of nodes including node i and its immediate neighbors N_i:

$$e_i = \sum_{\forall j,k \in \{N_i \cup i\}} a_{jk}/2$$

While the size of a network can be measured by the number of nodes, n does not give much information about distances within the network. Two measures of network distance are commonly employed: diameter ($d_{\max}$) and characteristic path length (L). If D is a matrix in which the off-diagonal elements d_{ij} give the minimum number of links that one would need to traverse to get from node i to node j, then the diameter of the network is:

$$d_{\max} = \max_{ij} d_{ij} \tag{4}$$

The characteristic path length is the average of all d_{ij}:

$$L = \frac{1}{n(n-1)} \sum_{\substack{\forall i,j \\ i \neq j}} d_{ij} \tag{5}$$

One of the important contributions of [3] was to describe a model that shows the "small-world phenomena" described by Travers and Milgram [39]. In small-world networks L increases roughly with $\log n$, whereas in regular lattice structures L increases linearly with n.

Finally we compare the degree assortativity in the test networks. Degree assortativity (r) in a network is defined in [40] as the extent to which nodes connect to nodes with similar degree. Formally assortativity is the correlation in degree for the nodes on opposite ends of each link [41]:

$$r = \frac{m^{-1}\sum_{i=1}^{m} j_i k_i - \left[m^{-1}\sum_{i=1}^{m}\frac{1}{2}(j_i+k_i)\right]^2}{m^{-1}\sum_{i=1}^{m}\frac{1}{2}(j_i^2+k_i^2) - \left[m^{-1}\sum_{i=1}^{m}\frac{1}{2}(j_i+k_i)\right]^2} \tag{6}$$

where m is the number of links in the network and j_i/k_i are the degrees of the endpoints of link i.

2.4. Results

Analysis of the IEEE 300 bus and EI network models clearly indicates that power grids have exponential degree distributions, as found in random graphs. We perform two statistical hypothesis tests to verify this relationship. Hypothesis 1 is that the synthetic networks have the same degree distribution as the empirical degree distribution in the Eastern Interconnect ($\Pr(k) \sim \Pr(k : \text{EI})$). Hypothesis 2 is that the synthetic graphs have a power-law degree distribution ($\Pr(k) \sim k^{-\alpha}$). We use a Kolmogorov-Smirnov t-test to evaluate these hypotheses. To find the power-law distribution fit parameters (α and $x_{\max}$) we use the method described in [42]. Table 2 shows results from these tests, as well as other measures of network structure. Figure 2 shows the degree distributions for the large graphs.

To summarize we find that the random graph nearly matches the degree distribution of the Eastern Interconnect, but the other synthetic networks do not. Similarly we find that the degree distribution of the Eastern Interconnect does not follow a power-law ($P_{KS} < 0.001$), whereas our PA graph aligns well with a power-law degree distribution. Figure 2 shows the degree distributions for the 49,597 node graphs. Because the degree distribution has a moderately fat tail there is a significant difference between the random graph and the larger power grid. A significant difference between the 300 node power grid and random graph was not observed.

A number of other results in Tables 1 and 2 are notable. The power grid graphs show a high degree of clustering, but not as high as the small-world graph. Given the differences in clustering and degree-distribution, it seems unreasonable to argue that the small-world model provides a good representation of power grids. Also the assortativity in the power grids is negative, whereas the small-world model shows a positive correlation. As expected, the larger preferential-attachment and random graph models show nearly zero assortativity.

3. The minimum-distance graph

The preferential attachment, small world, and random graph models have proven to be useful in representing a number of systems in which distance and cost are not particularly important. In the world-wide web for example, the cost of link-creation is independent of physical distance. In infrastructure systems, the physical cost of creating new links has an enormous influence on network structure. In power grids, gas pipelines, and water distribution systems (among others) link costs scale roughly linearly with geographic distance and hubs are infrequent. The minimum-distance graph described below represents these evolutionary dynamics in a fairly simple way and generates topolo-

Table 1. Statistical properties of 300 and 49,597 node graphs

Network	IEEE-300	Eastern Intercon-nection	Random	Random	Small world $p = 0.08$	Small world $p = 0.0882$	Preferential attachment	Preferential attachment
Nodes	300	49597	300	49597	300	49597	300	49597
Links	409	62985	409	62985	402	62906	409	62966
$< k >$	2.73	2.54	2.73	2.54	2.68	2.54	2.73	2.54
$\max(k)$	11	27	7	13	6	6	32	391
C	0.11	0.071	0.008	0.00004	0.26	0.27	0.008	0.0006
L	9.9	35.8	5.7	11.3	9.6	36.6	4.4	7.2
$d_{\max}$	24	96	12	26	24	96	9	18
r	-0.22	-0.11	0.044	-0.0012	0.034	0.12	-0.19	-0.024
Hyp. 1	-	-	do not reject	reject**	reject**	reject**	reject**	reject**
Hyp. 2	marginal	reject**	reject*	reject**	reject**	reject**	do not reject	do not reject
est. of α	3.5	3.5	3.5	3.5	3.5	3.3	2.49	2.88

* Significant at the 0.01 confidence level.

** Significant at the 0.001 confidence level.

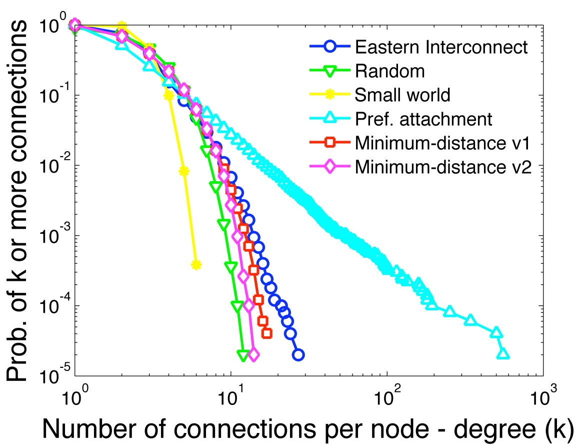

Figure 2. Degree distributions for 49,597 node graphs

gies with properties that are similar to those observed in power grids.

3.1. Simple minimum-distance graph

The following algorithm can be used to generate minimum distance graphs. Let M_a be the set of neighboring nodes for node a.

- For $a = 1 : n$
  1. Randomly generate planar coordinates for a (x_a, y_a) from a uniform distribution
  2. Generate approximately floor(m/n) links ($a \rightarrow b$) by iteratively selecting b to minimize the Euclidean distance between a and b.

$$\begin{aligned} \min_b \quad & (x_a - x_b)^2 + (y_a - y_b)^2 \\ \text{s.t.} \quad & b \notin M_a \end{aligned} \tag{7}$$

  3. Generate one additional link from Eq. 7 with probability $P = m/n - \text{floor}(m/n)$.

3.2. The minimum distance graph with bisection

To improve the fit slightly we propose a one-parameter version of the simple attachment kernel above. In many infrastructure systems, when a new node is created, the node can either be connected to an existing node through a new link, or it can be placed near and existing link, and become a bisecting node for that link. Consider for example the construction of a new town that needs to connect to the power grid. The town can either be connected via a transmission line or feeder to an existing substation, or a new substation could be build, which would bisect an existing transmission line. To modify the algorithm we introduce one parameter, the bisection cost: c_b. With this modification, the min-dist selection criteria (Eq. 7) is modified slightly as follows:

$$\begin{aligned} \min \quad & (C_1, C_2) \\ \text{s.t.} \quad & C_1 = \min_{b \notin M_a} (x_a - x_b)^2 + (y_a - y_b)^2 \\ & C_2 = \min_{i \in \{1 \dots m\}} d(a \rightarrow e_i) + c_b \end{aligned}$$

where $d(a \rightarrow e_i)$ is the distance between point a and the nearest point along line segment e_i, and c_b is an exoge-

Table 2. Statistical comparison of minimum distance graphs and power grid topologies - 49597 node graphs

Network	East. Int.	Min-dist	Min-dist-2 $c_b =$ 0.008
Nodes	49597	49597	49597
Links	62985	62985	62963
$< k >$	2.54	2.54	2.54
$\max(k)$	27	19	14
C	0.071	0.23	0.23
L	35.8	15.8	34.4
$d_{\max}$	96	38	83
r	-0.11	0.16	0.12

nously selected bisection cost. If C_1 is lesser, then the algorithm creates a new link $a \rightarrow b$. If C_2 is lesser then the new node bisects the existing link e_i.

3.3. Properties of minimum-distance graphs

Minimum distance graphs exhibit properties that are similar to those that we find in power grids. As found in the small world and power grid topologies the clustering coefficients are fairly high. As found in the random graph and the power grid, the degree distributions are exponential. As we find in power grids and regular lattices, the characteristic path lengths appear to scale linearly with n, rather than logarithmically. We continue to see positive, rather than negative, assortativity in the mid-dist graphs. In future work we will look for ways to produce graphs with anti-correlated node connectivity.

4. The electrical structure of power grids

In order to fully understand the structure of a power grid, one needs to know not only its topology, but also the structure that results from the physical properties that govern flow. To understand the electrical structure of a given power grid we need a measure of electrical connectedness, or conversely distance. Electrical distance has been used in a number of power systems problems [43, 44, 45, 46], although, aside from [43], electrical distance has not been used extensively in the context of structural network analysis. While electrical distance does not perfectly represent all of the ways in which components in a grid connect, it is a useful starting point for structural analysis. There are numerous variant measures of electrical distance, but one of the simplest is the absolute value of the inverse of the system admittance matrix (see. Eq. 8).

$$\mathbf{E} = |\mathbf{Y}^{-1}| \tag{8}$$

This electrical distance matrix, E with elements e_{ab}, gives the sensitivity between voltage and current changes for every node pair. Using this measure we can define a measure that is roughly analogous to node degree, but for a fully connected network with continuous weights for each node pair. To do so we define a measure of connectivity distance for each node a:

$$\overline{e_a} = \sum_{\substack{b=1 \\ b \neq a}}^{n} \frac{e_{ab}}{n-1} \tag{9}$$

This measure can be translated into a measure of electrical centrality by inverting the distance measure:

$$c_a = \frac{1}{\overline{e_a}} \tag{10}$$

A similar measure was first proposed in [23], though in a slightly different form. Figure 4 illustrates the electrical centrality of the IEEE 300 bus system.

It is possible to produce a graph representation of the electrical structure of the system from the electrical distance matrix. We keep the original n nodes, but replace the m links with the m smallest entries in the upper (or lower) triangle of $\mathbf{E}$. Thus we create a graph with size $\{n, m\}$ with links representing strong electrical connections rather than direct physical connections. The adjacency matrix of this new graph ($\mathbf{R}$) is defined as follows:

$$\mathbf{R}: \begin{cases} r_{ab} = 1 & \forall e_{ab} < t \\ r_{ab} = 0 & \forall e_{ab} \geqslant t \end{cases}$$

where t is a given electrical distance threshold, which we adjust to give 411 links. Figure 3 shows the electrical structure ($\mathbf{R}$) for the IEEE 300 bus test case (also first presented in [47]). Comparing the topology ($\mathbf{A}$, Figure 3) to the electrical structure ($\mathbf{R}$, Figure 3) shows a stark contrast between the electrical and topological structure of the test system. A similar structural difference is found by comparing the topology and electrical structure of the EI model. Table 3 summarizes the calculated metrics for the Eastern Interconnection $\mathbf{R}$ matrix. One can see that there are nodes with a very high degree. Clustering coefficient is also relatively high respect to the value obtained from the equivalent topological network. Both average shortest path (L) and diameter (d_{max}) hold low values indicating a strong electrical connectedness. Figure 5 shows the distribution of normalized distances (ccdf) obtained from the EI model. The topological distance distribution obtained from the $\mathbf{D}$ matrix is smooth and it has an exponential tail, whereas the electrical distance distribution ($\mathbf{E}$ matrix) has a different decay shape.

4.1. Electrical distances to load

Given the electrical distance matrix $\mathbf{E}$, it is possible to rank nodes by the amount of load/demand that is within a

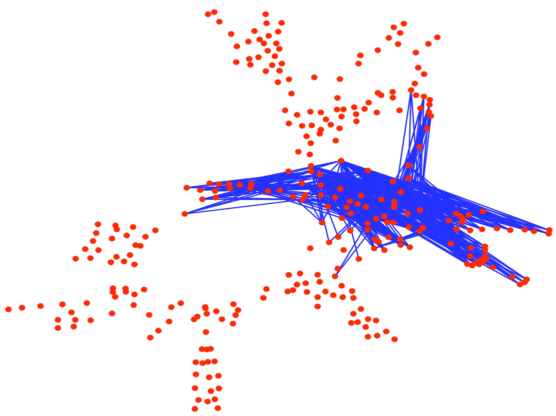

Figure 3. A graphical representation of the electrical topology of the IEEE 300 bus system, formed by replacing the 411 transmission branches with 411 shortest distance electrical links (the 411 smallest dij such that i > j).

Table 3. Topologically equivalent metrics for the EI reduced electrical distance matrix ($\mathbf{R}$).

Network	EI reduced electrical distance matrix*	EI topology ($\mathbf{A}$ matrix)
Nodes	1558 (48039 isolates)	49597
Edges	62984	62985
$< k >$	2.54	2.54
$\max(k)$	1473	27
C	0.65	0.071
L	1.97	35.8
d_{max}	3	96
r	-0.51	-0.11

(*) Metrics are calculated for the giant component.

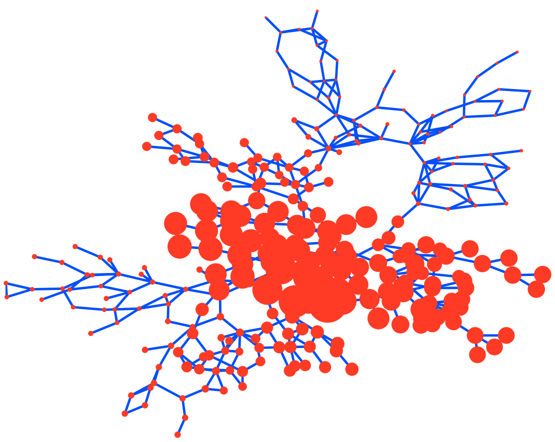

Figure 4. The IEEE 300 bus network with nodes/buses drawn with sizes proportional to their "electrical centrality" (Eq. 10).

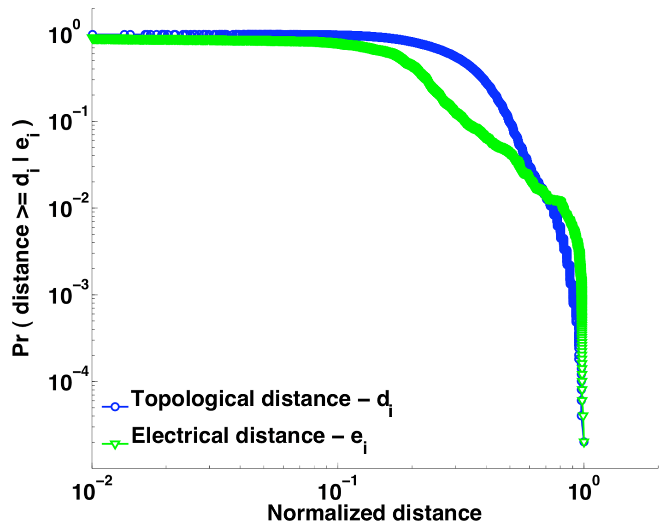

Figure 5. Cumulative probability distributions for electrical and topological distances (d_{ab} and e_{ab}) for the US Eastern Interconnect.

given electrical distance threshold. Such a measure of electrical distance to load enables additional insight to the electrical structure of power grids. In this section we determine the electrical distance to load in the approximately 4500 buses within the PJM controlled portion of the EI. From the electrical distance matrix, $\mathbf{E}$, we plot the percentage of the total system load that is reached as electrical distance increases from each bus. The resulting figures (6 and 7) enables the visualization of the electrical proximity to load.

It is clear that the network has several groups of nodes that are similarly distant from system load. By understanding the physical topology of the network we observe that nodes that are similarly distant from load are typically located close to one another. However, nodes that are electrically closest to load are not necessarily close, electrically or physically, to each other. This suggests that there are several hubs with a high electric centrality each serving different loads. The shoulder that occurs at roughly 70% of load is likely results from the fact that roughly 25% of the buses are separated from the rest of the system by a small group of transmission lines. Since a reasonably large fraction of the total system load is located in this remote portion of the network, this 25% of load shows up as a shelf in Figures 6 and 7. Typically, we are more concerned with the load reached at very short distances from each node rather than the distance at which most of load is reached.

The surface in Figure 7 shows that when nodes are sorted by the distance at which most of the load is reached, the distribution is relatively uniform. However, the variability of slopes on the bottom 60% of the figure suggest that there are several nodes that reach about 50% of load at a short distance while maintaining a large maximum distance.

Using these techniques, several analyses can be performed in future work. It would be useful to determine the exact load served by a particular generator, or rather the generation sources for a particular load in order to determine the environmental effects of consuming electricity. The relevant market for electricity consumption could also be determined. The location dependent market price of electricity is determined by the result of an optimization problem and it is difficult to determine price sensitivities to changes in load or generation levels. The electrical distance to load enables the simplification of the problem by reducing the relevant market for electricity consumption to a much smaller number of nodes. These carbon footprint and market power analyses are just a couple examples of the work that is possible with a better understanding of the electrical grid's network structure.

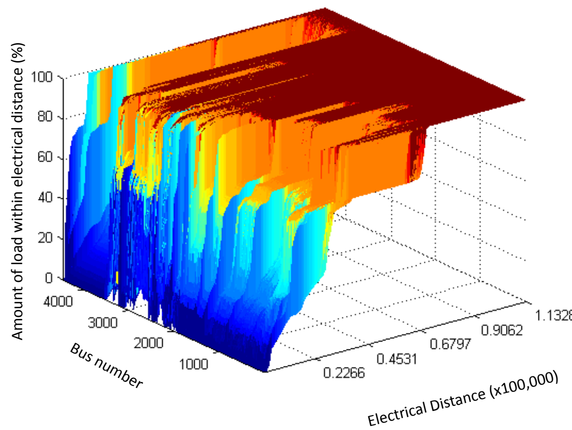

Figure 6. Electrical distances to loads for the PJM portion of the US Eastern Interconnect. In this figure buses are sorted by bus number in the model, which roughly groups buses by geographic proximity.

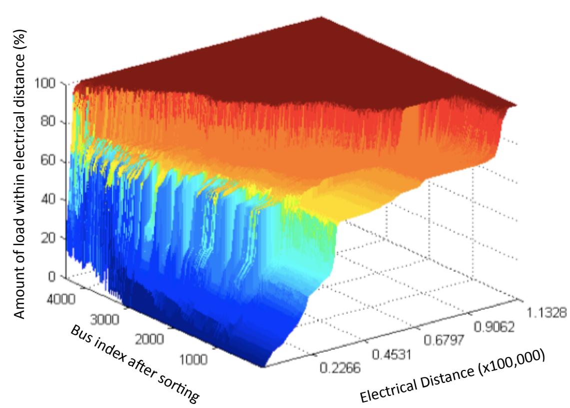

Figure 7. Electrical distances to loads for the PJM portion of the US Eastern Interconnect. In this version of the figure buses are sorted by the electrical distance at which 90% of the load can be reached.

5. Conclusions and implications for vulnerability analysis

In this paper we have presented a number of results that (1) characterize the topological structure of the US Eastern Interconnect, (2) illustrate a new method for creating synthetic power grid topologies, and (3) outline differences between the electrical and topological structure of power grids.

Regarding the topology of power grids we find that the IEEE 300 bus case and the EI have a clearly exponential degree distribution, showing that power grids are not Scale Free in topology. This is a valuable result because it allows us to confidently reject the hypothesis that the observed power-law distribution in blackout sizes [20] does not result from a power-law in grid topology. On the other hand, the electrical structure of power grids may provide some insight into the power-law in blackout sizes. An investigation of this remains for future work. Also we find significant differences between the EI model and an equivalently sized Small World network, indicating that the Small World model may not provide good insight for the study of power grids. Finally we find that adjacent nodes in the the EI and IEEE 300 systems have anti-correlated degrees (i.e. power grids are dis-assortative), which differs somewhat from the results reported in [27].

Having noted some unique properties of power grids, we propose a new graph-generating algorithm, the minimum distance graph, that produces random networks that are roughly similar to what we find in power grids (aside from the negative assortativity). In future work we plan to refine the min dist algorithm and use it to test the hypothesis that it provides a better representation of the structure of built infrastructure networks (power grids, gas pipelines, water distribution networks, etc.) then existing synthetic network models.

Finally, we propose several ways for representing the electrical structure of power grids, and find that this electrical structure is profoundly different than the topological structure. We suspect that these results will lead, in future work, to new tools for risk identification and mitigation. For example, we conjecture that measures of electrical distance to load will be useful in identifying buses that are particularly important to the stability of the grid as a whole, and therefore warrant additional hardening to random failure and directed attacks.

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