Estimating Dynamic Instability Risk by Measuring Critical Slowing Down

Paul D. H. Hines, *Member, IEEE*, Eduardo Cotilla-Sanchez, *Student Member, IEEE*, Benjamin O'hara, and Christopher Danforth

Abstract—Cascading failures typically involve a wide variety of power system dynamic phenomena, including cascading transmission line overloads, generator tripping, voltage collapse and/or rotor dynamic instability. Metrics that estimate proximity to critical points with respect to any of these phenomena could be useful as indicators of cascading failure risk. With the growing deployment of phasor measurement units (PMUs) in power systems, there is a rapidly increasing quantity of high-resolution, time-synchronized phasor data available to operators. Information in these data that could be formed into metrics of proximity to critical transition could be valuable to system operators who need to make timely, and costly, decisions to avert large blackouts. This paper provides preliminary evidence that time-series data alone, without intricate network models, can signal a pending critical transition in power systems. Our method is based on identifying "critical slowing down" in time series data. Results from a single machine stochastic infinite bus model and the Western US blackout of 10 August 1996 illustrate the utility of the proposed method.

Index Terms—Cascading failures, power system monitoring, power system reliability, synchrophasors.

I. Introduction

RELIABLE electricity infrastructures are vital to modern societies, but are notably susceptible to large, cascading failures. The disturbances of, for example, 14 August 2003 in North America [1], 4 November 2006 in Europe [2] and 10 November 2009 in South America [3] emphasize the continued risk associated with large cascading outages. Given that natural and human exogenous forces will occasionally result in component failures and that large blackouts contribute disproportionately to overall blackout risk [4], [5], there is a continuing need for new approaches to the identification of risks in power systems. Most, if not all, cascading failures involve several types of dynamic phenomena such as:

- transmission line outages due to contact with vegetation (thermal overload) or due to distance relay threshold crossings
- generator outages resulting from off-nominal conditions (e.g., over or under excitation)
- voltage collapse, or near voltage collapse
- generator rotor dynamic instability
- small-signal instability

   With the implementation of synchronized phasor measurement units (PMUs or synchrophasors) operators have

increasing access to large quantities of high-resolution, time-synchronized data. Methods that can turn these data into information about operating risk could dramatically increase the value of synchrophasor technology, and help operators to make better decisions about when or if to implement emergency operating procedures.

It is well known that the trajectory of eigenvalues (poles) in a power system, or in any dynamical system, can be used to predict critical transitions such as voltage collapse or dynamic instability [6], [7], [8], [9]. However, the precise measurement of eigenvalue trajectories in a large system requires accurate models and large quantities of sensor data. Power system failures sometimes progress across the boundaries of balancing authorities, where sensor data are aggregated. Furthermore, even within a balancing authority, cascading failures can progress more quickly than the communications and computational processes from which eigenvalues are calculated. Therefore there is a need for tools that can identify emerging risks without detailed, highly accurate, network models.

A number of methods for estimating blackout risk from phase-angle data exist in the power systems literature. Recent advances in the use of PMU data are described in [10], [11]. Dobson [12] describes a method for estimating angle differences between based on PMU measurements and circuit theory. Senroy [13] describes a method for measuring phase differences between groups of generators from time-series data. This paper also looks at trends in PMU data that could correlate to elevated blackout risk, focusing particularly on methods for identifying signs of critical slowing down (CSD) [14], which can be an early warning sign for bifurcation phenomena. Building on substantial literature on CSD, Scheffer et al. [15] describe methods for detecting proximity to transition in a variety of complex dynamical systems through the use of autoregression models. In this paper we apply the method described in [15], [16] to a two bus, single machine infinite bus power system model (SMIB) and data from the August 10, 1996 blackout in the Western North American Interconnection (WI).

II. Measuring critical slowing down

As described in [15], noisy systems that are being driven toward a critical point (e.g., a point of instability or oscillation) frequently show a decrease in the rate at which the system returns to equilibrium before reaching a point of critical transition. This phenomenon is commonly known as "critical slowing down," and was originally described in models of

emergent magnetic fields in ferro-magnetic materials [14]. Critical slowing down appears to signal proximity to a wide variety of critical transitions in large complex systems such as climate models before catastrophic climate change occurs [16] and the human body before an epileptic seizure [17]. Similar phenomena may be useful in signaling proximity to critical transition in power grid models as well.

Given this background, the goal of this work is to develop and test methods for identifying statistically significant signs of critical slowing down in power systems. In each test we will test the hypothesis that time-series voltage or phase-angle data show measurable evidence of CSD during times of elevated blackout risk.

In order to test this hypothesis we follow and build on the procedure defined in [16], which describes the use of autocorrelation (or autoregression) to test for criticality in global climate models. The following steps summarize the procedure used in [16], as adapted for this research, in which we seek to identify CSD in a time domain signal $x(t)$ (phase angle, $\theta(t)$, and frequency, $\omega(t)$).

1) Choose a window size within which to measure autocorrelation. This window should be large enough to include enough data to minimize the impact of spurious changes. Choosing a window that is several times larger than one period of the slowest oscillatory mode is generally a good approach. In this paper we use a 2-minute window size.

2) Filter the data in each window to remove gradual trends that are not the result of CSD (e.g., the slow change of phase angle at a bus due to a change in load). Following the method in [16] we use a high-pass filter based on a Gaussian Kernel Smoothing ($\hat{x} = x - GKS(x)$) function to remove trends slower than 0.1 Hz (SMIB) or 0.2 Hz (WI).

3) Choose a sample time-lag that will be used for the autocorrelation calculation. In order to obtain the autocorrelation coefficient for a 120 sec. window ending at time t, using a 1.0 sec. time lag, we use the following expression: $\rho(t) = \sum_{\tau=t-120}^{t} \frac{\hat{x}(\tau)\hat{x}(\tau-1)}{\sigma_{\hat{x}}^2}$, where $\rho(t)$ is the autocorrelation coefficient for the window that ends at time t and $\sigma_{\hat{x}}^2$ is the variance in the signal within the window.

4) Test for statistically significant increases in $\rho(t)$ using the nonparametric Kendall's τ coefficient. Kendall's τ tests for serial dependence in a signal, against the null hypothesis that the signal is random. This test is performed at 30-second intervals.

5) Finally, in order to corroborate the findings from Kendall's τ, we measure the power spectral density (PSD) of $\hat{x}(t)$ using a Welch spectral estimator[18], which will show an increase in low-frequency components if the system is slowing down.

The following sections describe the application of this method to a single machine infinite bus model and data from the August 1996 blackout in Western North America.

III. SINGLE-MACHINE STOCHASTIC-INFINITE-BUS MODEL (SMSIB)

In this section we describe a modified version of the classical single machine infinite bus (SMIB) model, and outline ways in which critical slowing down appears in this system. In our two-bus model we gradually increase stress in the system by linearly increasing the amount of power generated by the generator. Also the infinite bus voltage (V_2) is modified by adding noise to the voltage source (sink). We model the noise as a bandwidth-limited Gaussian white noise, where the voltage at Bus 2 is:

$$V_2(t_k) = 1.0 + \sigma_V \mathcal{N} \ \forall t_k \in \{0.00, 0.01, 0.02, \cdots\}$$

(where σ_V is the standard deviation of the noise and $\mathcal{N}$ is a Gaussian random variable), and $V_2(t)$ between the discrete time steps, t_k, is interpolated using a cubic spline. In this paper we use $\sigma_V = 0.01$ for the noise magnitude. The noisy infinite bus simulates the effect of exogenous, small, voltage flicker in the larger system to which the generator is connected[19]. A similar model, with noise in the generator power rather than the infinite bus voltage, is explored in [20].

In our model, the generator is located at Bus 1, with terminal voltage $V_1 = |V_1| e^{i\theta_1}$. It is modeled as a classical round rotor, lossless generator that produces $P_e(t)$ electric power as a result of $P_m(t)$ mechanical forcing. The generator has a constant field voltage magnitude ($E_q = 1.1$ p.u.) behind a synchronous reactance ($X_d = 0.1$). The rotor dynamics are governed by the classical swing equation [?] with $P_e(t)$ subject to the network equations for this specific circuit:

$$P_m(t) = P_e(t) + D\dot{\delta}(t) + M\ddot{\delta}(t) \tag{1}$$

$$P_e(t) = \frac{|E_q|V_2(t)}{X_d + X_{12}} \sin(\delta(t)) \tag{2}$$

where $\delta(t)$ is the machine rotor angle, relative to the phase angle of the infinite bus ($\theta_2 = 0$), D and M are machine damping and inertia constants and X_{12} is the reactance of the transmission line between the two buses. The trajectories of δ and θ_1 are calculated using a variable step size, implicit Runge-Kutta differential algebraic equation solver [21], [22]. The output data from the DAE solver (most notably $|V_1(t)|$ and $\theta_1(t)$) are subsequently sampled at 30Hz to obtain simulated outputs from a phasor measurement unit. The machine has a damping constant of $D = 1.5$ p.u. and an inertia constant of $M = 3$ p.u. The two buses are connected via a single transmission line with impedance: $Z_{12} = j0.1$ p.u.

From (2) it is clear that the SMIB model becomes unstable when P_m reaches $P_m = \frac{|E_q|V_2(t)}{X_d + X_{12}}$. The poles of the swing equation (1) can be found from the eigenvalues of the linearized state matrix:

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ \frac{-E_q V_2}{M(X_d + X_{12})} \cos \delta(t) & -\frac{D}{M} \end{bmatrix} \tag{3}$$

Figure 1 shows the poles of this system for different values of δ. As δ increases, such as results from increasing the mechanical power into the machine (P_m), the imaginary portions of the eigenvalues of $\mathbf{F}$ converge toward the origin, until shortly
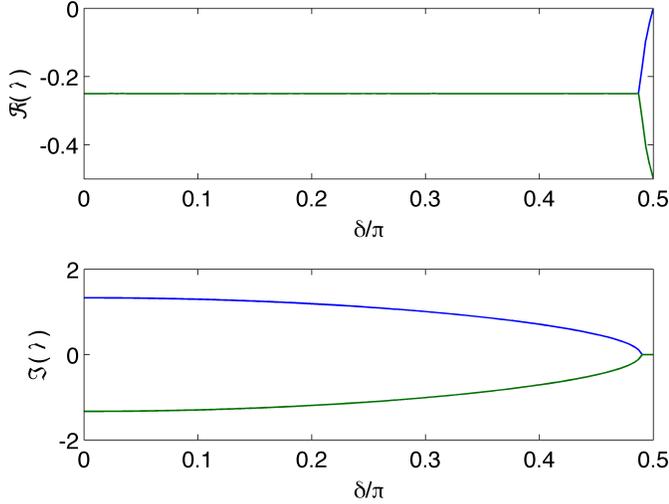
Figure 1. Real (above) and imaginary (below) components of the poles of the linearized state matrix $\mathbf{F}$ (1) at different values of δ. In our model P_m increases linearly over time and, for steady state conditions, δ is a direct function of P_m, from (2). Thus the critical frequency of the system gradually decreases as P_m increases, which means that lower frequency elements in the noise will be amplified and recovery times will increase.
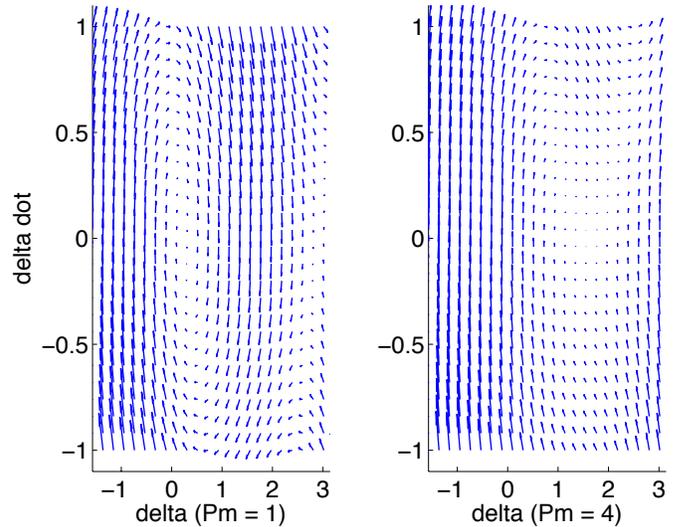

Figure 2. Vector field diagram showing the basin of attraction and the forcing vectors for the two-bus system for a lightly loaded case ($P_m = 1.0 p.u.$, left) and for a heavily loaded case ($P_m = 4.0 p.u.$, right). For the lightly loaded case the basin of attraction is fairly narrow, whereas in the heavily loaded case the basin includes a large area with small force vectors. In this case small perturbations could result in substantial deviation from the point of equilibrium and recovery times will increase.

before $\delta = \pi/2$, they meet at the real axis. At this point the poles of the system lie on the real axis with the right-most eigenvalue quickly crossing into the right half plane, leading to system instability. Critical slowing down is apparent in several ways. As the dominant frequency of the system decreases the relaxation time will increase, which is a symptom of critical slowing down. Also, as the poles near the point at which the system transitions from oscillatory to exponential, small changes in δ or in the noise (V_2) will substantially change the dominant frequencies in the system, which will result in a wider range of frequencies being accentuated. This phenomena is sometimes apparent as flicker, another sign of critical slowing down. Both of these phenomena can be observed in the power spectral density of the signal as an increase in the power of lower-frequency components. These phenomena will also result in an increase in autocorrelation.

Another way to visualize critical slowing down is to look at a vector field showing the basin of attraction for SMIB model if it is perturbed from its steady state conditions (Fig.

A. Stochastic models for the infinite bus

In future work we plan to model the noise in V_2 with a Ornstein-Uhlenbeck mean reverting process (OU)[23], [24]. The OU process models a continuous signal $u(t)$ that wavers randomly but tends to revert to a mean level μ_u. It is given by the stochastic differential equation (SDE)

$$
\begin{aligned}
du(t) = & \ \eta(\mu_u - u(t))dt + \sigma_u dW_t \\
u(0) = & \ u_0
\end{aligned}
$$

where η is the speed of mean reversion, σ_u is the short term standard deviation and W_t is a Wiener process.

Using the Ito interpretation [24] we can rewrite the solution of the SDE as a Gaussian model where given u_0, the value of $u(t)$ is normally distributed with mean $\mu = \mu_u + (u_0 -$

$\mu_u)e^{-\eta t}$ and variance $\sigma^2 = \frac{\sigma_u^2}{2\eta}(1 - e^{-\eta t})$. In order to provide the DAE solver with a continuous process, we interpolate the realization of the WGN vector ($\mu = \overline{V_2} = 1.0\angle 0°$ p.u. and $\sigma^2 = 0.0001$ p.u.) with the method of cubic splines.

B. Two-bus model results

Fig. 3 shows the results that emerge from the two bus model as it is forced toward the maximum power transfer limit. Providing evidence in support of the hypothesis, the autocorrelation in the phase angle data at Bus 1 increases notably minutes before the system hits the point of maximum power transfer. Kendall's τ (lower panel in Fig. 3) indicates that this increase is statistically significant. Furthermore the power spectral density of the signal (middle panel) shows substantial increases in low-frequency signal power as the system approaches the critical transition. The variance in the signal similarly increases steadily throughout the simulation.

IV. WESTERN INTERCONNECT BLACKOUT OF AUGUST 1996 (WI)

On August 10, 1996 a long sequence of events resulted in the separation of the North America Western Interconnection into five sub-grids and the interruption of electric service to 7.5 million customers. Reference [25] describes the sequence of events leading up to the blackout, and [26] provides a detailed analysis of the power system dynamics during the event. In [25], the WSCC (now Western Electricity Coordinating Council, WECC) disturbance study committee provided about 10 minutes of measured bus voltage frequency data from the Bonneville Power Administration territory, up until just after the point of grid separation. In order to test for CSD in these data, the printed frequency charts were scanned and
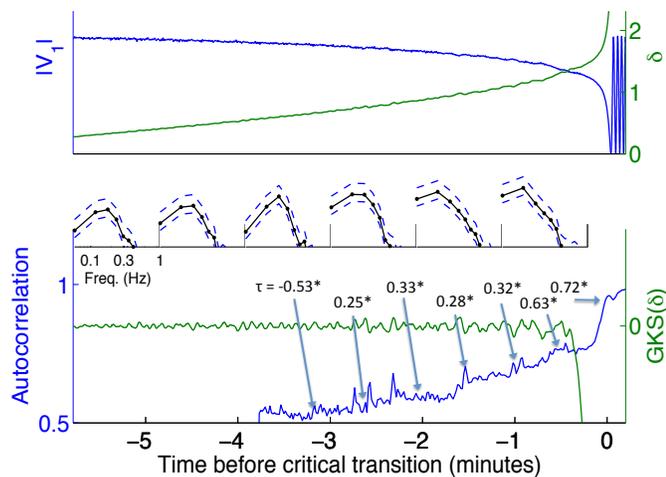
Figure 3. Evidence of critical slowing down in a two-bus (SMSIB) power grid model being driven toward the point of maximum power transfer. The middle panel shows the power spectral density of the signal (the phase angle at Bus 1, θ_1) for vertically projected time intervals. The dashed curves indicate the 95% confidence margins for the PSD estimate. Numbered arrows illustrate the measure of Kendall's τ coefficient at 30-second intervals. *Indicates that τ is statistically significant at the $P < 0.05$ level, which indicates a statistically significant increase (or decrease) in autocorrelation.

translated into a numerical time series and the tests described above were repeated. As was found with the two-bus model, autocorrelation in the frequency signal increases significantly as the critical transition approaches, as does the power spectral density of low frequency changes (See Fig. 4). Kendall's τ shows that the increases in autocorrelation are statistically significant.
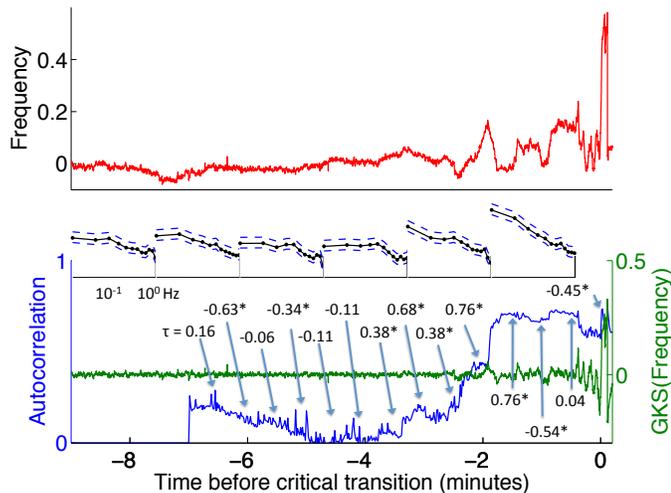

Figure 4. Evidence of critical slowing down in the frequency as measured at the Bonneville Power Administration, immediately before the blackout of August 10, 1996. As in Fig. 3, the low-frequency components of the signal (middle panel) increase notably immediately before the transition occurs.

V. CONCLUSIONS

In this paper we describe a method for testing for critical slowing down in power systems and provide evidence that CSD is present as power systems approach a point of dynamic

instability. The results indicate that critical slowing down could be useful in identifying operating states with unusually high dynamic risk. Unlike traditional stability methods, the proposed statistical approach uses only high-resolution time data and could therefore be useful even if SCADA/EMS systems fail, so long as the operator has access to time synchronized phasor data.

While this measure alone is not a perfect indicator of cascading failure risk, it may prove useful, when combined with other indicators of power system cascading failure risk, in the development of an aggregate measure of cascading failure risk.

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AUTHOR BIOGRAPHIES

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Paul D. H. Hines (S'95, M'07) is an Assistant Professor in the School of Engineering at the University of Vermont. He is also a member of the Carnegie Mellon Electricity Industry Center Adjunct Research Faculty and a commissioner for the Burlington Electric Department. He received the Ph.D. in Engineering and Public Policy from Carnegie Mellon U. in 2007 and the M.S. degree in Electrical Engineering from the U. of Washington in 2001. Formerly he worked at the US National Energy Technology Laboratory, where he participated in Smart Grid research, the US Federal Energy Regulatory Commission, where he studied interactions between nuclear power plants and power grids, Alstom ESCA, where he developed load forecasting software, and for Black and Veatch, where he worked on substation design projects. His main research interests are in the areas of complex systems and networks, the control of cascading failures in power systems and energy security policy.

Eduardo Cotilla-Sanchez is a Ph.D. student in Electrical Engineering in the School of Engineering at the University of Vermont. He earned the M.S. degree in Electrical Engineering from the University of Vermont in 2009. His primary research interests include complex systems and evolutionary computation, in particular, the application of these disciplines to power and energy systems.

Benjamin O'Hara is an undergraduate student in Mathematics in the College of Engineering and Mathematical Sciences at the University of Vermont.

Christopher Danforth is an assistant professor in the Department of Mathematics and Statistics at the University of Vermont.