

# Evaluating the Impact of Modeling Assumptions for Cascading Failure Simulation

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**Abstract**—Because of the complicated combination of mechanisms that combine to produce large power system failures, the simulation of cascading failure requires some modeling assumptions. In this paper we compare three models of cascading failure in electrical power systems under various assumptions. In the first, we combine dynamic generator models with a DC power flow network model, and use time-delayed (memory) relays to simulate branch failure. In the second, we simulate cascading with the use of sequential power flow calculations. In the third, we simulate cascading using a simple topological contagion model. The results indicate that the dynamical and the quasi-steady state (QSS) simulations show substantial agreement, whereas the topological model differs significantly. We also find that the extent of the agreement between the dynamical and the QSS model largely depends on the way in which branch failures occur.

**Index Terms**—Cascading failure, Linear dynamic model, Quasi-steady state model, Transient behavior.

## I. INTRODUCTION

THE electrical power system industry is of vital importance to economic, social and national security. It is one of the few industries in which the set of affected stakeholders may be said to encompass all of modern society. It is however exposed to a significant risk of cascading failure that may propagate throughout the system [1]. A cascading failure is defined by NERC as: “The uncontrolled successive loss of Bulk Electric System Facilities triggered by an incident (or condition) at any location resulting in the interruption of electric service that cannot be restrained from spreading beyond a predetermined area.” [2]. The loss of service may result in great costs to society, such as those observed due to the August 2003 blackout in North America [3].

The minimization of cascading risk is therefore a continuing area of research in both academic and industrial settings. Despite these efforts, salient fundamental questions on the nature of cascading failure in power systems remain unanswered. These include how to make modeling

assumptions and comparisons between models. In this paper we test some frequent modeling assumptions using traditional and a novel metrics that compare failure paths.

The power system cascading failure problem is multi-dimensional and the robust determination of relevant factors has yet to be achieved. These factors include those mentioned in Section II of this paper. Due to the nature and operation of power systems it is extremely challenging to determine the correct assumptions for the simultaneous increase of modeling fidelity and decrease of computational complexity.

The remainder of this paper is organized as follows: In Section II we discuss some of the factors that need to be considered in power system cascading failure modeling and provide a classification of models within the literature. In Section III three distinct models are introduced in order to determine the impact of some of the modeling assumptions described in Section II, which may affect the outcome of cascading failure. Comparisons of the cascading failures produced by each model are presented in Section IV and a discussion is given in Section V.

## II. CLASSIFICATION OF CASCADING FAILURE MODELING ASSUMPTIONS

The modeling of cascading failure in power systems can be viewed as a multi-dimensional problem. This is due to the nature of the multiple interactions of underlying technological, physical and policy restrictions that can affect the operation of the system during a cascade. In this section a number of possible dimensions are discussed.

### A. Probabilistic vs. Deterministic failures

Probabilistic models of cascading outages are common in the literature [4]-[6], due to the uncertain nature of many of the component failures. There are, however, models that deterministically fail overloaded components [5], [6]. Although we consider only deterministic models in this paper, this limitation can be easily circumvented by adding support for probabilistic failure to any of the deterministic models. This will be described in future work.

### B. Adherence to Kirchhoff's Laws

It has been contended that the endogenous failure of components to a system perturbation is the result of changes in the components loadings. Therefore, models of cascading failure in power systems must be based on approximations to the physics that govern the load redistribution. Thus as a general rule the models must incorporate at least an

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approximation of Kirchhoff's Laws as the main method of loading redistribution. Despite this requirement some models of cascading failure in the literature use topological characteristics to describe the failure of components [7], [8]. Adapted from [9] one such model is discussed in this paper. Whereas some models are based purely on Kirchhoff's laws [4], others use a combination of topological and Kirchhoff's laws for component failure [5], [6].

### C. Simultaneous vs. Single Line Failures

During a cascading event multiple lines may become overloaded. As it has been addressed in [8], the order in which lines are failed can influence the progression of the cascade. In particular, it is important to consider whether the model allows for simultaneous failure of lines or only one failure at a time. In this paper, we test different assumptions regarding the sequence in which lines fail.

### D. Steady State vs. Transient Modeling

A model can be classified on whether, or not, it incorporates transient behavior of the power system as transmission lines and generators are failed. A model that includes transient behavior would include generator dynamics and the possible dynamic simulation of load demand [6]. Our experiments in Section IV also benchmark this distinction.

### E. Persistent Memory of Previous States

In many cascading failure models the current state of the system is used in order to determine the risk of component failure. We propose that the risk of failure of components is not only dependent on the current state of the system but also on previous operating states. Consider, for example, two transmission lines that have the same characteristics and are at the same steady state power flow. We can imagine that both lines have a temperature at which heating will cause sagging and increase the risk of failure. If they previously are at different steady state power flows and temperatures they will arrive at the new equilibrium temperature at different times and therefore will have different associated risk of failure.

### F. Operator Response

The manner in which operators (or automated control schemes) respond to a cascading failure event is also a salient influence in the progress of the event. Remedial action schemes can affect the size of the resulting blackout positively or negatively. For example, the European blackout of 2006 was largely the result of human error [9]. The human part of the system is probably the most difficult to model, due to the vast uncertainty involved in modeling the behavior of human agents. In this paper we model one type of human/automated reaction to cascading failure events by adjusting whether or not generators are dispatched after line tripping or islanding.

### G. DC vs. AC transmission system

Cascading models may also be classified based on whether they compute the power flows in the transmission system using a full AC analysis or using the DC load flow assumptions. The use of DC load flow assumptions is common practice in the literature due to the speed up that can be achieved and thus allowing a larger number of cascading failures to be examined [4], [5]. AC models do exist [6];

however, to the best of the authors' knowledge, the translational advantage of increased accuracy of power flows into the probability distributions of cascading failures at various operating points has not yet been tested. Another important feature of AC modeling is the inclusion of voltage stability and reactive power limits in the model. These mechanisms play a significant role in large cascading failures [10]. The consideration of voltage stability, however, requires new mechanisms of load modeling that are beyond the scope of this work.

## III. CASCADING FAILURE MODELS

In this paper we compare three distinct models of cascading failure. The first two are based on the DC power flow assumptions while the third is based on a topological model. This section describes the properties of these three models.

### A. Linear Dynamic Model

The Linear Dynamic (LD) model reproduces the cascading failure mechanisms by combining generator dynamics with the DC power flow assumptions for network flows. The mathematical formulation of the model is based upon a system of Differential Algebraic Equations (DAE) [11]. While the assumptions of this model are restrictive, the computational speed at which it runs allows for a large number of cascading failures to be analyzed while still increasing modeling fidelity over other cascading failure models that use solely a DC power flow ([4], [5]).

The dynamical aspect of the LD model consists of generators that are controlled through second order governor models. These governors introduce transient behavior in the system as lines are switched off and generator or loads become disconnected. While not considered in this work, the set of state variables currently implemented for the generators may be used to produce frequency dependent failures of the generators, thus leading to cascading failures more inline with their real world limitations. This effect could easily be replicated in other simplified cascading power system models while still producing relatively short running times (when compared to slower AC models such as the Manchester model).

The set of algebraic equations is composed by the usual power injection and angle equations under the DC power flow assumptions:

$$P = B\theta \quad (1)$$

where  $P$  is the set of power injections or withdrawals from the system,  $\theta$  are the angles at each bus and  $B$  is the system's Kirchhoff's matrix [12].

The differential equation associated with each generator is the classic swing equation [13]:

$$M\ddot{\delta} = P_E - P_m - D\dot{\delta} \quad (2)$$

where  $\delta$  is the vector of rotor angles of the generators,  $M$  is the inertia of the machine and  $D$  is the damping constant.  $P_E$  and  $P_m$  are the electric power exported from the generator and the mechanical power of the generator respectively.

The LD model also incorporates a simple second order

governor model so that it may be able to respond to changes in loading due to generator or loads being tripped out of the system.

Another set of memory variables is associated with the power transmission lines. These variables give the associated memory value to each line, which is integrated over the entire cascade time span and increases or decreases depending on its current state and the flows in the line. Each variable correlates to the temperature of the components in the transmission line, however it does not necessarily represent only the temperature of the line. It may also be considered to incorporate other effects associated with failure risk that are compounded during the operation of the component such as wear and tear and sagging. The differential equation for the memory variable is given as:

$$\dot{T}_i = F_i^2 - k_i T_i \quad (3)$$

where  $T_i$  is the memory variable for line  $i$ ,  $F_i$  is the flow on the line and  $k_i$  is a time constant associate with the line.

For each line there is a cut-off value for the memory variable,  $T_i^C$ , which is associated with its equilibrium value if the line was loaded at rate A for an infinite amount of time. The constant  $k$  is set so that starting from a reference value, which is taken as zero, the variable will reach the cut off,  $T_i^C$ , after ten minutes if the line is loaded at its rate B flow. This cut-off mechanism is deterministic and due to the nature of equation (2) no line can be tripped if the flow in the line is not at some point in time above its rate A value. The initial memory values,  $T_i^0$ , are obtained by assuming that the system was at an equilibrium state.

The LD model also includes a stopping criterion when no lines have failed in the previous 30 minutes. We assume that this would be long enough for the operator to respond to a set of failures.

### B. Quasi-Steady State Model

The Quasi-Steady State (QSS) model was created as a complement to the LD model in order to investigate assumptions of models that rely solely on the steady state operation of the system. There are no dynamics in the model other than the switching of lines due to overloads.

This model is also based on DC power flow assumptions and it is dispatched using an optimal power flow. Initially, the model tries to find a solution where no load shedding occurs and all lines remain at their rate A values. However, if no solution is found, the system will be dispatched so that the rate A but not the rate B limits of the lines can be reached.

Lines are tripped deterministically when they are above their rate B values. The main distinction between the LD and QSS models is the manner in which the lines are failed. In the LD model the memory variable takes into account the entire history of the line loadings during the cascade while the QSS does not. For the QSS model only the current state of the system is relevant for the cascade.

### C. Topological Contagion Model

A number of cascading failure research articles are based on experiments on topological models of the power grid [14]. The

cascading mechanisms of many of those models resemble the “domino” effect, where a given component of the network will fail if topologically connected components have failed as well. In the field of complex networks, these are also denominated “contagion models”. In this paper, we describe results from a topological contagion model (TC) which is very similar to the one proposed in [15]. Each node represents a transmission line and these nodes can be active nodes (failed) or inactive nodes (healthy). Note that this representation is dual to the classic convention of node/bus and link/transmission line.

The following algorithm describes the discrete dynamics that govern our topological contagion model:

1. Assign contagion thresholds to each transmission line. These values are drawn from a random distribution  $f(\phi)$  such that  $f$  is a probability density function. A transmission line,  $i$ , will switch from inactive to active status when the proportion of neighboring transmission lines that are active is greater than  $\phi$ .
2. At the start of the simulation, activate the transmission lines that belong to a contingency set, and initialize the rest of transmission lines to inactive status.
3. At each time step, randomly and asynchronously update the status of each transmission line according to the comparison between its own contagion threshold  $\phi(i)$  and the current number of active neighbors.
4. Repeat Step 3 until we reach the end of the simulation.

## IV. EXPERIMENTS AND RESULTS

The three cascading models were simulated on the IEEE 39-bus test system. The goal was to compare the differences in cascading failures resulting from the same exogenous events. The system was initially dispatched to be N-1 secure and then subjected to N-2 and N-3 contingencies. The active power flow line limits were chosen according to the following formula:

$$rateA = cB_{ij} \quad (4)$$

where  $B_{ij}$  is the susceptance of the line and the constant  $c$  represents where the limit thresholds are so that at the optimal dispatch, no N-1 contingencies are present in the QSS model. Its value in this case was 0.2127775. The rate B limit of the line was chosen to be 1.1 times the value of rate A for all lines. Unless otherwise stated the constant  $k$  from Eq. 3 was set at 0.0029. This value gives a time to failure of ten minutes (when a line is operating at its rate B value) from a cold start. That is, from a starting point of zero for all values of  $T$ , the line reaches its equilibrium rate A,  $T_c$ , in ten minutes.

For each simulation, we recorded the initial and final state of the system, as well as the timing in which faults occurred.

For simplicity, within these experiments we define a cascade as a continuation of failures after the initial set of exogenous events.

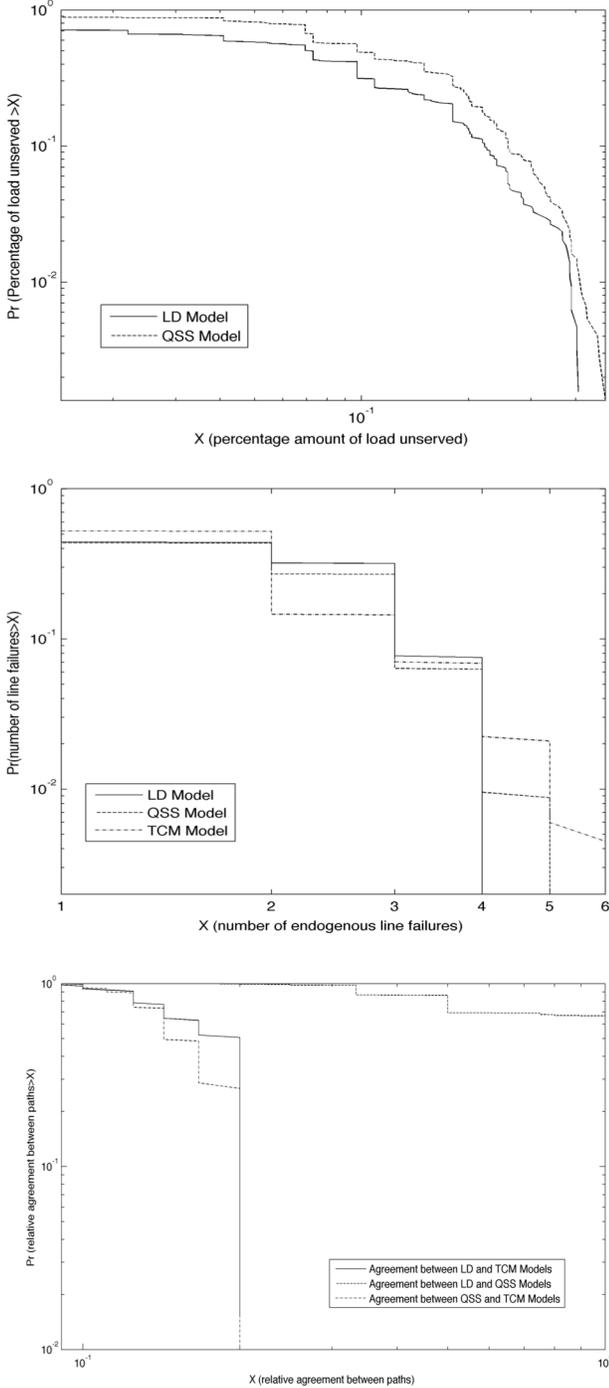


Fig. 1. CCDFs for the three metrics for each model. Top panel is the percentage load lost in the LD and QSS models. The second panel is number of endogenous line outages in the cascade for the three models. The last panel is for the relative agreement between paths for the three models.

The resulting cascades were compared using three metrics:

1. Cascade size, measured in number of line failures.
2. Blackout size in percentage of unserved load.

3. The relative agreement between cascade paths.

The experiment metrics were designed in order to adhere to the desire of a system operator to identify the size of a potential cascading failure. This is estimated with the combination of the first two metrics and the path of failure given by the third metric.

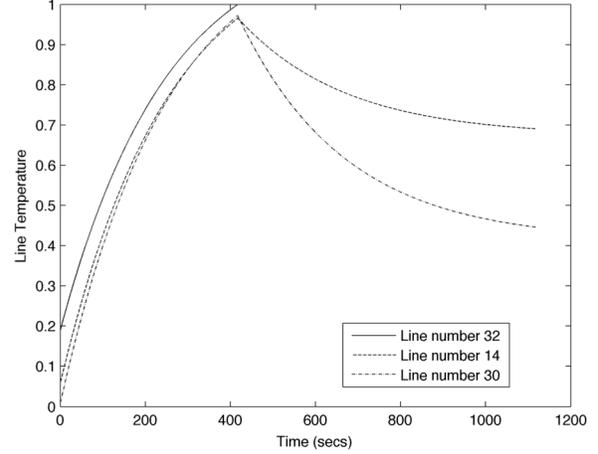


Fig. 2. Normalized memory of three overloaded lines where the cascade stopped due to Condition 2.

The relative agreement of cascade paths,  $R(m_1, m_2)$ , is defined as follows. If models  $m_1$  and  $m_2$  are both subjected to the same set of exogenous contingencies:  $c = \{c_1, c_2, c_3, \dots\}$ .  $R(m_1, m_2)$  measures the average agreement in the set of dependent events that result from each contingency in each model. If contingency  $c_i$  results in the set of  $A_i$  dependent branch failures in model  $m_1$  and the set of  $B_i$  dependent branch failures in  $m_2$ ,  $R(m_1, m_2)$  is defined as,

$$R(m_1, m_2) = \frac{1}{|C|} \sum_{i=1}^{|C|} \frac{|A_i \cap B_i|}{|A_i \cup B_i|} \quad (5)$$

We also measure the distribution of  $R(m_1, m_2)$  for cases where the set of contingencies is defined as individual elements of the set of  $C$ .

For the N-2 contingency scenario, all possible sets that could occur were run for both the LD and the QSS model. It was observed that in not a single case did the exogenous events cause a continuation of failures of other components in the LD model when no such events occurred in the QSS model. This outcome suggests that for the transmission failures being modeled here, the transient behavior introduced by the generators did not influence the modes of cascades of the system. Therefore, the steady-state line flows are the most predictive factor of cascading failures modes in these two models.

For N-3 contingencies only the scenarios that produce a cascade in the QSS model were investigated within the LD and TC models.

### A. Comparison of the LD, QSS and TC models

In the first experiment the models were run on the benchmark system described above. The complementary cumulative distribution functions (CCDF) for the various metrics are shown in Fig. 1. In the top panel we compare the QSS and LD models regarding the amount of load unserved after the completion of the cascade. The panel shows that the LD model gives a higher probability of less load being shed than the QSS model. Similarly, in the second panel of Fig. 1, the probability of a certain number of line failures in the LD model were lower than in the QSS model for the majority of sizes.

The reason for the discrepancy between the models is based on two assumptions within the LD model that are not present in the QSS model:

1. Time constraints on the continuation of the cascade in the LD model.
2. The constraint that only one line may fail at a given time.

In terms of the first constraint it is observed that approximately 62% of the cascades stop at some point during the cascade whereas they would have continued should they be given more time. Of these cascades, 56% were simply a single additional endogenous event in the QSS model with the LD model not initiating any dependent cascading failures. The remaining 6% of the cascades stop at some point (in the LD model) after an initial sequence of endogenous events.

For the cascades that did not stop due to Condition 1, it was observed that for 29% of the total number of cascades there was agreement between the QSS model and the LD model. For the remaining 9% the LD model produced shorter cascades in terms of endogenous line failures. It was also observed that in all cases where these differences occur, multiple line overloads were present and therefore there were multiple possible paths of cascading failure. The converse however is not true since some cascades with multiple possible paths were shown to have a final end state, which was the same in both models.

We conjecture that the temporal nature of failure in the LD model was the result of this discrepancy. That is, the failure of one line at a time may have reduced the flows in other stressed lines bringing the system back into a normal operating condition. This in turn caused the cascade to stop. These cascades show that it is important to assume the temporal failure order in cascading failure models. An example is shown in Fig. 2 with an scenario where three lines were overloaded after the initial N-3 contingency. Upon the first line reaching its temperature limit and failing, it caused a redistribution of flows that reduced the flows in the other overloaded lines and a subsequent reduction in the temperature of those lines.

In the case of the TC model the distribution of line failures was similar to the other two models. However, as shown in the third panel of Fig. 1, the relative agreement of paths between the TC and the other two models was much lower than between the QSS and the LD model. This result is expected as the both the QSS and LD model failures are based on

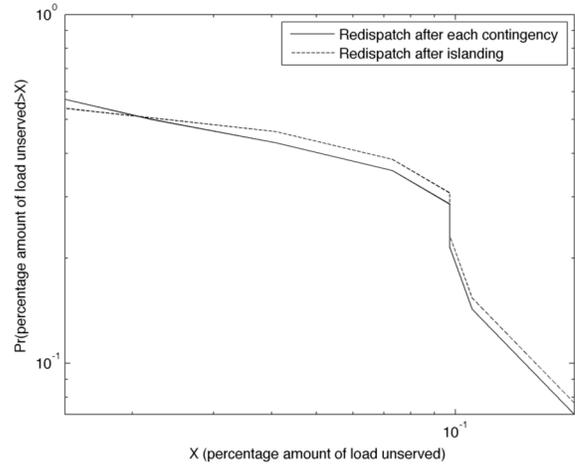


Fig. 3. CCDF of load unserved for redispatching the system after a single failure event and only after islanding for the LD model.

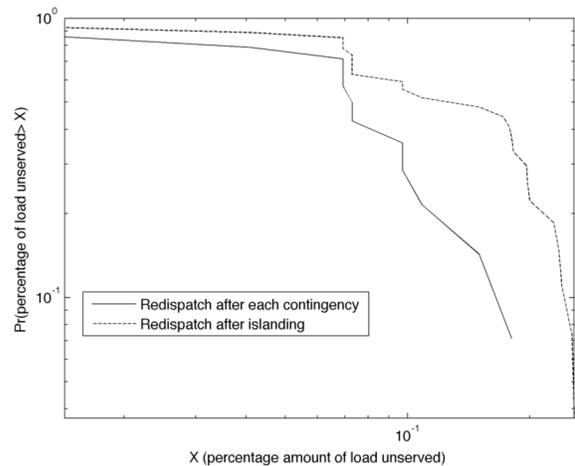


Fig. 4. CCDF of load unserved for redispatching the system after a single failure event and only after islanding for the QSS model.

Kirchhoff's laws while the TC is simply a topological nearest neighbor model. Since the redistribution due to Kirchhoff's Laws affect potentially almost all other elements in the system, the resulting cascades should look very different to the TC model failures.

### B. Assumptions regarding the order in which lines fail

Since we observed in the previous section that the temporal nature of line switching could affect the resulting cascade it would seem prudent to investigate the role of line switching order for a system with multiple overload on cascading failures. Here, the condition of failing all overloading lines in the QSS model was relaxed and several failure sequences were used to compare with the original QSS model and the LD model.

Our criteria to choose sequences is similar to those in [16]:

1. The line with the largest flow as a percentage of its Rate A value was failed first.

2. The line with the highest absolute flow in terms of MWs was failed first.
3. The line with the lowest flow as a percentage of its Rate A value was failed first.
4. The line with the lowest absolute flow in terms of MW's was failed first.

It was found again that there was a general agreement between the distinct QSS models, with the conditions described above. The mean agreement variable,  $R(m_1, m_2)$ , between cascades for each rule governing the failure sequence, 1 to 4, and the original QSS model were approximately 0.98, 0.97, 0.98 and 0.95 respectively. As expected, the divergent paths between the sequences assumptions were found in cases where multiple line overloads occurred in the original QSS model. It is however again noted that multiple line overloads does not necessarily lead to such a divergence.

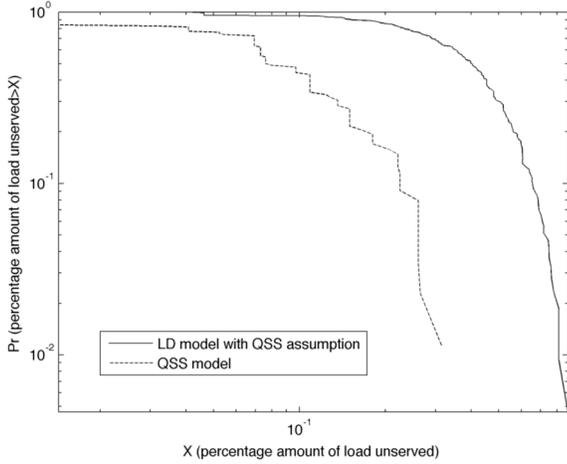


Fig. 5. CCDF of load unserved for the LD model with the QSS assumption of failure.

In comparison to the LD model, the sequences showed a low agreement, mostly due to the fact that the LD has a time constraint placed on whether a subsequent line failures is allowed or not. If we take out the time-related stopping cascades from the set we observe that there is a much higher agreement between the models. The path agreement variables are 0.9149, 0.8661, 0.9086 and 0.9843 for failure assumptions 1 to 4 respectively. This suggests that while the LD replicates none of the rules stated above, the use of any of them will increase agreement for Condition 2 cascades.

### C. Assumptions on generation dispatch

In the original set of models the system was redispatched after each line contingency. In normal operation this would not be the case therefore a representation of the actions of the operator need to be included. The models simulated so far feature an ideal optimal redispatch after a line contingency. To investigate how the operating actions may change the conditions of cascading failure the assumption that the generators were redispatched immediately was relaxed. After a line contingency the generation and load demand were fixed

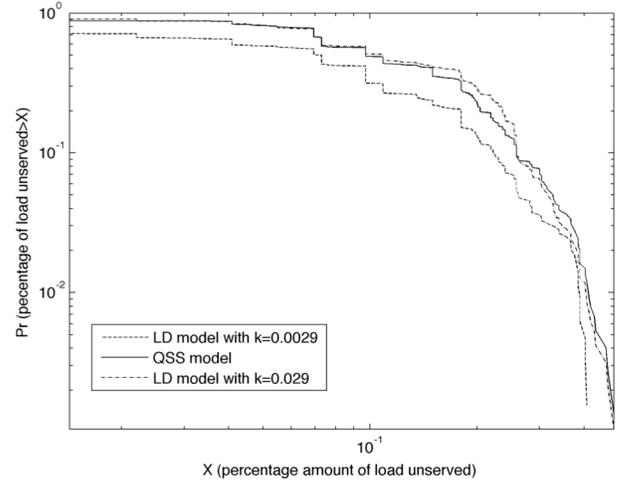


Fig. 6. CCDF of blackout sizes of the LD model with differing constants  $k$ . Two values are plotted against the QSS model.

Table 1. The relative agreement of the LD and QSS models for increasing values of  $k$ .

$k$	$R(m_1, m_2)$
0.0029	0.3536
0.0058	0.4271
0.0146	0.4402
0.0292	0.5949

to their previous values until an islanding event occurred and the system was then dispatched again to optimally serve the load. We found that for the N-2 contingencies that had no redispatch there was an increase the number of cascading failures in the QSS by threefold.

The CCDF of the percentage of load lost depending on the methods of dispatch is given in Fig. 3-4 for the LD and QSS models respectively. These figures illustrate that there is not only a larger set of N-2 events that can create a cascading failure but also that the probability of larger events in terms of MW lost is greater for the new dispatch assumption if a cascade does occur.

### D. Non-linearity of the models and failure assumptions

In order to investigate the possible interactions of model complexity and the modeling assumptions, the LD model was modified so that the failure of a line was determined in a similar manner to the QSS model.

The LD model has more fidelity than the QSS model to the operation of power systems, after each component failure. We are, therefore, able to test if increase in fidelity of the model necessarily results in more accurate cascading failure dynamics. The increase of the model fidelity, with the steady-state assumptions of cascading failure, cause the model to produce results that are not aligned to any of the models described above. The results for the LD model with this assumption are large cascades with concurrently large amounts of load shed. The agreement function between this instance of the LD model, the original LD model and the QSS model are almost zero. The above results show an important lesson in the formation of cascading failure models. In particular, an increase in the model fidelity that aims to bring

the model more inline with power system operation should also cause a reevaluation of the conditions on which lines are supposed to trip.

The reason for the large discrepancy is the generator and other dynamics of the system as a line fails the generators angles and the bus angles change and oscillate. This causes the power flow through a line to temporally swing above its rated value thus the line trips out. In the QSS model this dynamic behavior is not observed and therefore the QSS model gives shorter and more reasonable results. The CCDFs of the blackout sizes for the LD model with the QSS assumption failure are given in Fig. 5. These are shown in comparison to the QSS model for the same contingencies.

#### E. Change of memory model parameters

The value of the constant  $k$  was increased in order to observe the effect of increasing how fast the memory variable ( $T$ ) responds to changes in line power flows would affect the characteristics of the cascades. We conjectured that increasing the  $k$  value would increase the similarities between the QSS and the LD models, as more cascades that were stopped due to Condition 1, described above, would continue in the allotted time. For an increase in the  $k$  of 100% from its original value this was also observed to occur. The results show greater agreement with the QSS model while there was no difference in the paths between these LD models where the termination was the result of Condition 2 in both for the lower value of  $k$ . Similar results were observed after changing the time constant to 500% and 1000% of its original value. Fig. 6 shows the CCDF of two values of  $k$ . Here it is seen that the higher value is in more agreement with the QSS model. Table 1 gives a list of the relative agreement between the models as  $k$  increases. These results seem to suggest a convergence of the models as  $k$  is increased.

### V. DISCUSSION

In this paper three models of cascading failures in power systems were examined and comparisons were made of various assumptions that can be introduced into these models.

The results show that the two Kirchhoff models (LD and QSS) generally agree. When they differed it was largely the result of differences in the speed in which the transmission lines tripped. The results show that the model results converge as the time constant of the LD model was decreased. We also observed that the agreement between cascades that terminated naturally in the LD model did not change with the time constants. This suggests that changing the time constants is akin to speeding up the LD model. Therefore if the LD model was run for long periods of time, good agreement would be achieved with the QSS model. Discrepancies still exist between the two models; this is, when there are cases of multiple line overloads however this could be compensated by the introduction of rules for the order of failing overloaded lines. It is therefore suggested that the main assumption that guides the evolution of both models is Kirchhoff's laws. In the case of the TC model bad agreement was observed and there seems to be no modeling assumptions that could be adapted or relaxed in this model to achieve an agreement.

We observed that the inclusion of Kirchhoff's laws is not a sufficient condition for agreement of the models, as demonstrated in the case of the LD model with the QSS model assumption on line failure. Where the line was tripped after its power flow was greater than a cut-off value. In this case the transient dynamics of the LD model determined the evolution of the system more than the steady state redistribution of flows after contingencies.

Finally, we found that the actions of the operator can lead to substantial changes in the distribution of cascading events. However, these changes were qualitatively similar amongst the results provided by the models based on Kirchhoff's laws.

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